Strategic Game

* N (set of players)
* A (set of actions)
* Preferences

Stable Strategy: No player wishes to deviate from chosen strategy  
Dominant Strategy: Utility is greater than all other strategies  
Weakly Dominated: Utility is equal or less than others; doesn’t have to exist in mixed  
Strictly Dominated: Utility is less than others  
Nash Equilibrium: Every player can choose a single best strategy  
Mixed Equilibrium: Includes probability distribution of all strategies

Ui() = SUM [Ui(A) \* (A)] over all strategies  
 (A) = PROD [probability that player i would use strategy A] for all i

Correlated Equilibrium: For player 1, SUM [(A,B)\*(U1(A,B) \* U1(A’,B)] >= 0  
 For player 2, SUM [(A,B)\*(U2(A,B) \* U2(A,B’)] >= 0  
 Find a distribution where all of these are satisfied

Strategy Profile: At NE iff strategy profile > 0 implies that each player minimizes cost

Prisoner’s Dilemma

* U(Q) = U(Q) = -1/2 -4/2 = -2.5
* U(C) = U(C) = 0/2 -3/2 = -1.5
* Stable Strategy (NE): Both players confess

|  |  |  |
| --- | --- | --- |
| Payoff | Quiet | Confess |
| Quiet | -1, -1 | -4, 0 |
| Confess | 0, -4 | -3, -3 |

Matching Pennies

* U1(H) = ½ - ½ = 0
* U1(T) = - ½ + ½ = 0
* U2(H) = - ½ + ½ = 0
* U2(T) = ½ - ½ = 0
* No dominant strategy (Utility)
* No stable strategy (Equilibrium)
* MIXED  
  U2(pi) = pi2(H)\*U2(H) + pi2(T)\*U2(T)  
  U2(H) = -1\*pi1(H) + 1\*pi1(T)  
  U2(T) = 1\*pi1(H) + -1\*pi1(T)  
  ----------------------------------------  
  -pi2(H) + (1-pi2(H)) = pi2(H) – (1-pi2(H))  
  -2\*pi2(H) + 1 = 2\*pi2(H) – 1  
  2 = 4\*pi2(H)  
  pi2(H) = 1/2

|  |  |  |
| --- | --- | --- |
| Payoff | Head | Tail |
| Head | 1, -1 | -1, 1 |
| Tail | -1, 1 | 1, -1 |

Coordination Routing Game

* ISP’s compete over peering points; prefer to hijack the other ISP’s existing networks rather than using their own, but this adds significant extra load to that network
* U(HP) = - ½ - 4/2 = -2.5
* U(P) = - 2/2 – 5/2 = -3.5
* NE is (HotPotato, HotPotato)
* HotPotato has higher utility, and it is the only stable strategy

|  |  |  |
| --- | --- | --- |
| Payoff | HotPotato | Planned |
| HotPotato | -4, -4 | -1, -5 |
| Planned | -5, -1 | -2, -2 |

Load Balancing Game

* The load on a machine is the sum of the weights of its jobs
* The cost to the player is the same as the load in the machine it chooses
* Pure NE: each player chooses lowest cost with rest fixed
* Randomized strategy: Pij (prob that task I assigned to machine j)  
  n tasks, m machines, each task has weight and each machine has speed  
  execution time = Wi / Sj  
    
  EXPECTED LOAD =

Second Price Auction

* Given how much each player values the item (“valuation” V)
* U(i) = V(i) – bid(i) if player i wins, else 0
* Truthful NE: b(i) = V(i)
* Highest valuation always wins

Tragedy of Commons

* U(i) = traffic(i) – constant\*avgTrafficOfOthers
* When N >= constant, utility is maximized by taking as much of resource as possible

Cournot Game

* Infinite strategy set
* C = cost per unit offered
* R = supply of units
* Xi = unit of traffic sent by ISP i
* Best Response (i) = if (Xi <= 2-c):

((2-c) – Xi) / 2)  
else:

0

* Solve BR for each X to get NE

Coordination Game

* U(Player1) = 2\*(A)\*2(A) + 1\*1(B)\*2(B)
* NE occurs when no player wishes to change distribution (stable)

|  |  |  |
| --- | --- | --- |
| Payoff | Bach | Stravinsky |
| Bach | 2, 1 | 0, 0 |
| Stravinsky | 0, 0 | 1, 2 |

Battle of Sexes

* Ui(Prob) =
* U1() = 1(A)\*U1(A) + 1(B)\*U1(B)  
  U1(A) =   
  U1(A) = U1(B) >>> 22(A) = 2(B)  
  2(A) = 1/3, 2(B) = 2/3

Inspection Game

* V: product value
* H: inspection cost
* G: worker cost
* W: wage

|  |  |  |
| --- | --- | --- |
| Payoff | Inspect | Not Inspect |
| Shirk | 0, -h | w, -w |
| Work | w-g, v-w-h | w-g, v-w |

X = 1(S), Y = 2(I)  
P1: 0\*Y + w\*(1-Y) = (w-g)Y + (w-g)(1-Y)  
Y = g/w  
P2: -h\*X + v-w-h\*(1-X) = -w\*X + (v-w)\*(1-X)  
X = h/w

Congestion Game

* All have at least one pure Nash Eq

WHAT ARE CONVEX AND CONCAVE FUNCTIONS AND WHERE ON EARTH DID THEY COME FROM?

Sperner’s Lemma (rainbow triangles)

* This is super confusing and I don’t understand why we are coloring triangles